Normalized Transverse Current Distributions of Microstrip Lines on Anisotropic Substrates

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Abstract —**The normalized transverse current distributions are obtained for microstrip lines on anisotropic substrates and their dependence on ϵ_y^*, w/h, and the anisotropy ratio AR is explained. These distributions are classified into five cases. There are the cases in which the normalized transverse current distribution can be approximated by that obtained already for the cases of isotropic substrate.**

I. Introduction

DIELECTRIC anisotropy is present in many substrate materials used for microwave integrated circuits. This anisotropy occurs either naturally in a material or is introduced during a manufacturing process. Alexopoulos [1] summarized the problem of anisotropy using about 100 references and also gave new results.

It seems that the quasi-TEM analysis of microstrip lines on anisotropic substrates has been thoroughly investigated by many researchers both theoretically and experimentally [2]–[14].

Microstrip CAD requires accurate and reliable information on the dispersion behavior. This dispersion analysis has been studied by many researchers (see [1], [15]–[28], and references therein). However, there were significant discrepancies between many computed results even for the cases of isotropic substrates [18], [28]. Recently, Kretch and Collin [25] pointed out that there are also discrepancies between several computed results for the case of anisotropic substrates. To calculate accurately the dispersion characteristics requires expressing the current distributions on the strip accurately with a minimum number of basis functions [16], [27], [28].

The normalized longitudinal and transverse current distributions have already been obtained for the single microstrip lines of isotropic substrates and have been expressed in closed form [27]. Using these expressions and the spectral-domain analysis [16], the previous paper calculated the effective relative permittivities with a high degree of accuracy and tabulated those [28]. Recently, the present authors showed the normalized longitudinal and transverse current distributions on the coupled microstrip lines of isotropic substrates [30].

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An attempt will be made to tabulate the results with a high degree of accuracy for use as a numerical "standard" in the cases of anisotropic substrates. In the spectral-domain analysis used in calculating those, the choice of the basis functions for the current distributions is important for numerical efficiency. If the first few basis functions approximate the actual unknown current distributions reasonably well, the necessary size of the matrix can be held small for a given accuracy of the solution, so that CPU time can be saved. However, the literature determining current distributions is sparse, even for the microstrip line on isotropic substrate, as indicated in [27]. Recently, Shih *et al.* [29] proposed a full-wave analysis based on conformal mapping and variational reaction theory. The frequency dependences of current distributions were revealed for the cases of isotropic substrates. It is the first time that those characteristics were reasonably obtained for wide ranges of frequency. At lower frequency, the results were in good agreement with those [27] obtained using the same method as in present paper. The results of effective relative permittivities [29] were in good agreement with those tabulated in [28]. For an example of the current distribution for the cases of anisotropic substrate we may cite the paper by Sherrill and Alexopoulos [24] although it deals with the finline/strip configuration.

The present article shows the normalized transverse current distributions obtained using the method derived by Denlinger [15] and a Green's function technique [7], [9], [11] with concern for efficiency in obtaining dispersion characteristics for the cases of anisotropic substrates. The normalized longitudinal current distributions for the case of anisotropic substrates are not shown here because they can be approximated by the results for the cases of isotropic substrates [27].

II. Calculating Procedure

Fig. 1 shows the open microstrip line structure, assumed to be uniform and infinite in both the x and z directions. The infinitesimally thin strip and the ground plane are taken as perfect conductors. The structure is divided into two regions, corresponding to the air and the dielectric structure. It is also assumed that the substrate material is lossless and that its permittivity tensor and permeability

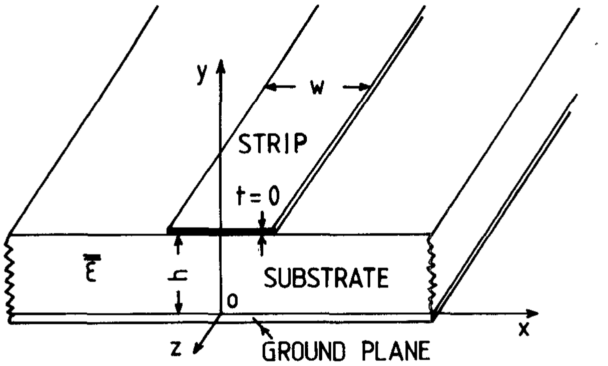

Fig. 1. Configuration of a microstrip line on anisotropic substrate.

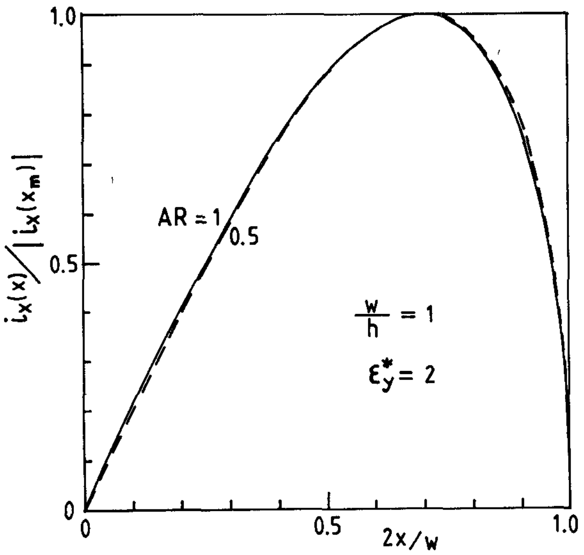

Fig. 2. Normalized transverse current distributions on the strip for the cases of $w/h=1$, $\epsilon_y^*=2$, $AR=0.5$ and 1.

are

$$\bar{\bar{\epsilon}}=\epsilon_0\begin{pmatrix}\epsilon_x^* & 0 & 0\\ 0 & \epsilon_y^* & 0\\ 0 & 0 & \epsilon_z^*\end{pmatrix} \tag{1}$$

and μ_0, respectively. Here ϵ_x^*, ϵ_y^*, and ϵ_z^* ($=\epsilon_x^*$) denote the relative permittivities in the directions of the principal dielectric axes of the substrate material, respectively.

Let the total longitudinal current be denoted by $I\exp(-j\beta(f)z)$, with the quantity $I/v(f)$ denoted by Q. Here $\beta(f)$ is the phase constant ($=\omega/v(f)$, $\omega=2\pi f$), $v(f)$ ($=c/\sqrt{\epsilon_{\text{eff}}^*(f)}$) is the phase velocity, $\epsilon_{\text{eff}}^*(f)$ the effective relative permittivity at the frequency f, and c the velocity of light in free space. We can use the following approximate expression [15] for obtaining the transverse current distribution on the strip:

$$i_x(x)=-j\omega(\operatorname{sgn}x)\cdot\int_0^x\{\sigma(x)-\epsilon_{\text{eff}}^*(0)\sigma_0(x)\}\,dx\,e^{-j\beta(f)z} \tag{2}$$

where

$$\operatorname{sgn}x=\begin{cases}-1, & x<0\\ +1, & x>0.\end{cases}$$

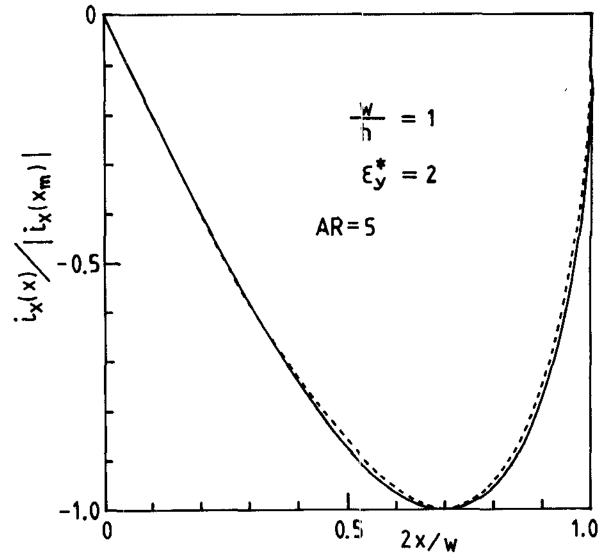

Fig. 3. Normalized transverse current distributions on the strip for the case of $w/h=1$, $\epsilon_y^*=2$, $AR=5$. ——, Present method. -----, Approximate formula for the case of isotropic substrate for $w/h=1$ [27] (with a negative sign).

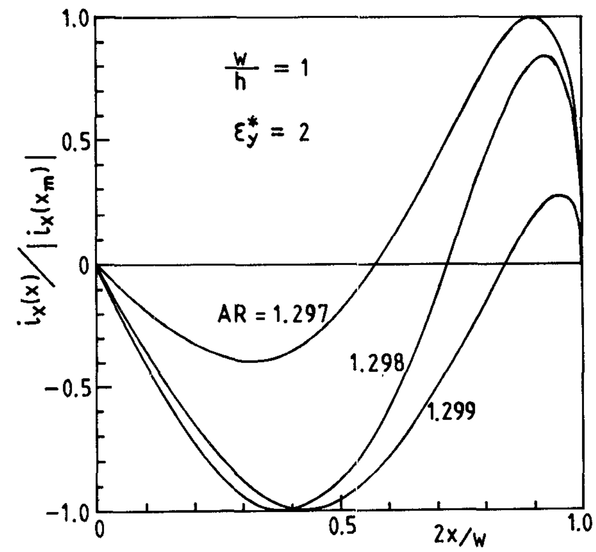

Fig. 4. Normalized transverse current distributions on the strip for the cases of $AR=1.297, 1.298, 1.299$ when $w/h=1$ and $\epsilon_y^*=2$.

Here $\sigma(x)$ denotes the charge distribution on the strip for a given total charge per unit length Q and $\sigma_0(x)$ denotes the charge distribution on the strip of the microstrip line without substrate for a given total charge per unit length $Q/\epsilon_{\text{eff}}^*(0)$. In the present article, $\sigma(x)$, $\sigma_0(x)$, and $\epsilon_{\text{eff}}^*(0)$ were calculated with a high degree of accuracy by using the Green's function technique [7], [9], [11] for various w/h, ϵ_y^*, and anisotropy ratio AR ($=\epsilon_x^*/\epsilon_y^*$).

III. Results

Fig. 2 shows the transverse current distributions $i_x(x)/|i_x(x_m)|$ on the strip normalized to $|i_x(x_m)|$ at the extremum point $x=x_m$ for the cases of anisotropy ratio $AR=0.5$ and 1 when $w/h=1$ and $\epsilon_y^*=2$. Good agreement is seen between the two curves. Next, we calculated

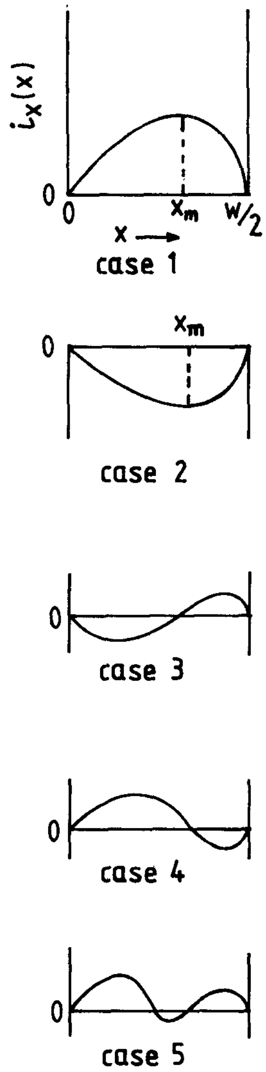

Fig. 5. Classification of normalized transverse current distributions of microstrip lines on anisotropic substrates.

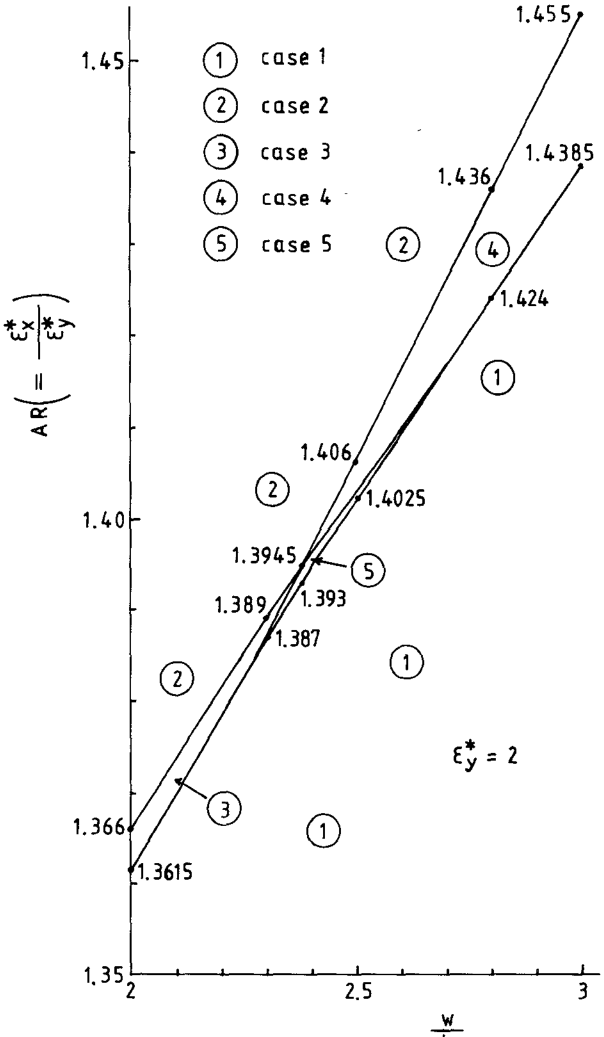

Fig. 6. Relations of anisotropy ratio AR and shape ratio w/h at the boundary between the five cases shown in Fig. 5 ($\epsilon_y^* = 2$).

$i_x(x)/|i_x(x_m)|$ for the case of $AR = 5$ and obtained an interesting result. This result is shown in Fig. 3 and has a negative value. This negative value was never obtained in the case of isotropic substrates [27]. Comparing Figs. 2 and 3, the curves are almost symmetrical with respect to the abscissa.

We calculated the current distributions for the cases of $0.5 < AR < 5$. Fig. 4 shows the typical results with both positive and negative extrema. Fig. 4 suggests that there is a transition region in the process changing from the curve only with positive extremum to the curve only with negative extremum when increasing the value of the anisotropy ratio AR.

Therefore, the normalized transverse current distributions were calculated for several cases with various w/h, ϵ_y^*, and AR. It was found that the current distributions can be classified into five cases, as shown in Fig. 5. Fig. 6 shows the region for each case when $\epsilon_y^* = 2$. The figures written in Fig. 6 denote the values of the anisotropy ratio AR at the boundary between two distribution forms. We obtained the distribution form of case ③ for $1.2955 \leqslant AR \leqslant 1.3$ when $w/h = 1$ although it is not shown in Fig. 6.

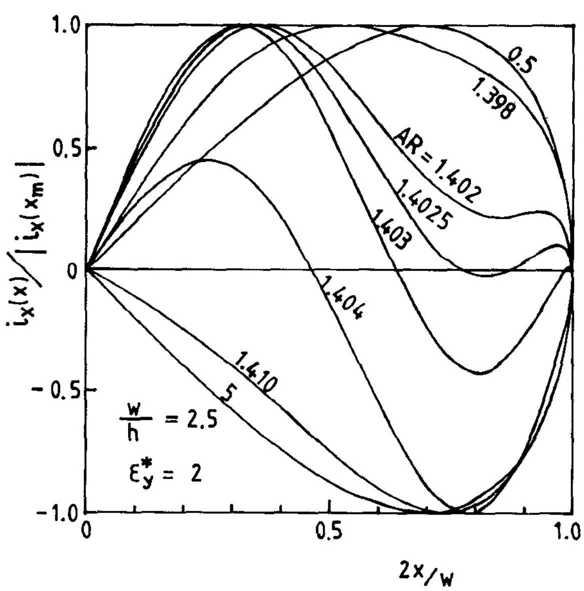

Fig. 7. Transition process in which the distribution form changes from case ① to case ② when increasing AR ($w/h = 2.5$ and $\epsilon_y^* = 2$).

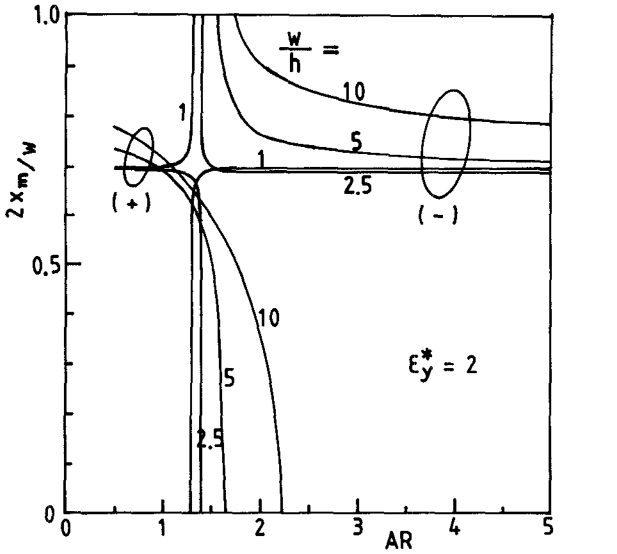

Fig. 8. Positions of the positive and negative extrema for the cases of $w/h = 1$, 2.5, 5, 10 when $\epsilon_y^* = 2$. (+): positive extrema. (−): negative extrema.

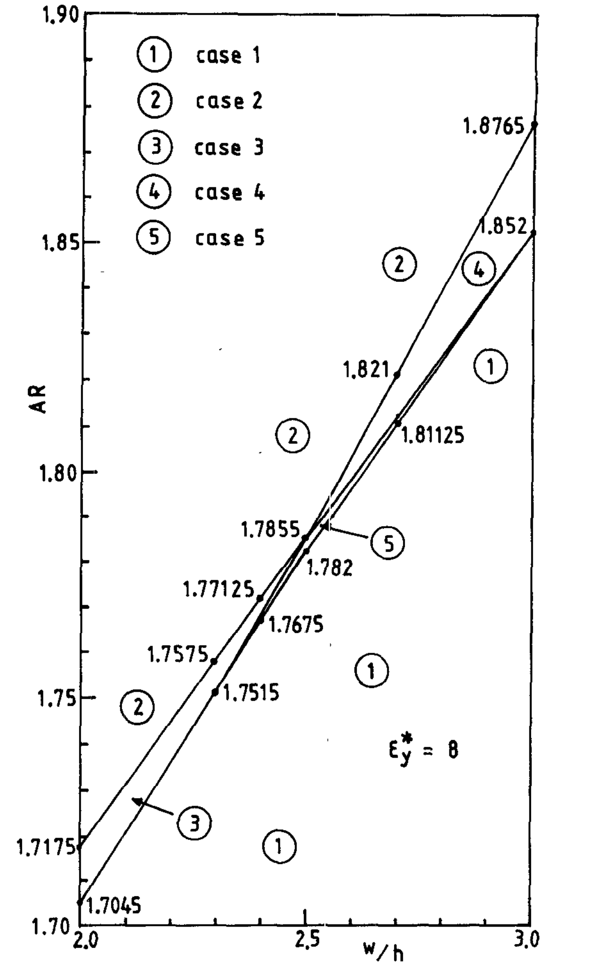

Fig. 9. Relations of anisotropy ratio AR and shape ratio w/h at the boundary between the five cases shown in Fig. 5 ($\epsilon_y^* = 8$).

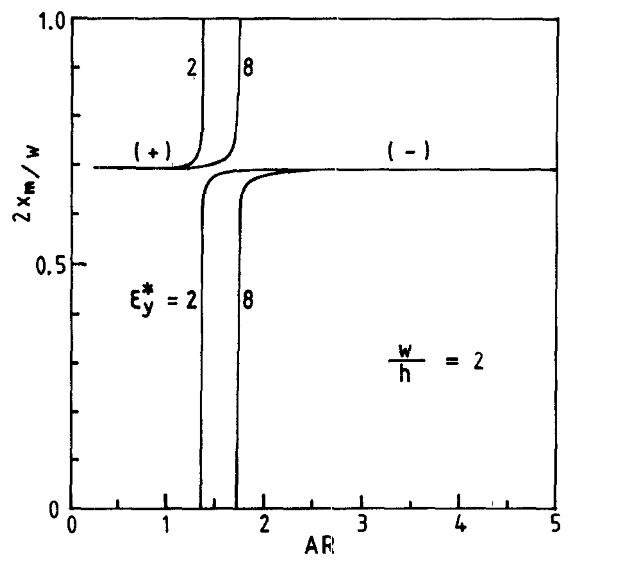

Fig. 10. Comparison of the positions of the positive and negative extrema for the cases of $\epsilon_y^* = 2$ and 8 when $w/h = 2$. (+): positive extrema. (−): negative extrema.

Fig. 7 shows the curves of the normalized transverse current distributions for the cases of various AR when $w/h = 2.5$ and $\epsilon_y^* = 2$. From Fig. 7, we can know the transition process in which the distribution form changes from case ① to case ② when increasing the value of AR.

Fig. 8 shows the positions of the positive and negative extrema for the various cases when $\epsilon_y^* = 2$.

Fig. 9 shows the region for distribution forms when $\epsilon_y^* = 8$. It is seen that the transition region shifts upward by about 0.36 in the value of AR from that for $\epsilon_y^* = 2$.

Fig. 10 compares the positions of the positive and negative extrema for the cases of $\epsilon_y^* = 2$ and 8 when $w/h = 2$. Excluding the transition regions, good agreement is seen between the two cases. This property holds for the cases of different ϵ_y^* when the same w/h is used, although not shown here. It is meaningful to say that the curves of normalized transverse current distributions are almost indistinguishable, as shown in Fig. 2, for the cases of different ϵ_y^* and AR, when w/h has the same value, as that at which the values of $2x_m/w$ exist on the flat part in the curves of $2x_m/w$ versus AR. For example, these cases correspond to the flat part in Figs. 8 and 10.

This teaches us that the closed-form expression for $i_x(x)/|i_x(x_m)|$ for these cases can be approximated by that for the case of isotropic substrate with $w/h =$ same value. The dotted line shown in Fig. 3 denotes the result obtained using the closed-form expression of $i_x(x)/|i_x(x_m)|$ derived in the previous paper [27]. Good agreement is seen between the theoretical and approximate curves.

IV. Conclusion

The normalized transverse current distributions have been derived for open microstrip lines on anisotropic substrates, and their dependence on ϵ_y^*, w/h, and AR has been explained. It has been shown that these distributions can be classified into five cases. It has been found that the curves of normalized transverse current distributions are almost indistinguishable for the cases of different ϵ_y^* and AR when w/h has the same value as that at which the values of $2x_m/w$ exist on the flat part in the curves of $2x_m/w$ versus AR. In these cases, we can approximate the normalized transverse current distribution by the closed-form expression for the case of isotropic substrate with $w/h =$ same value. Good agreement has been seen between the theoretical and approximate curves. Using these current distributions, the dispersion characteristics of microstrip lines on anisotropic substrates are being performed using spectral-domain analysis.

The current distributions depend on a frequency, as revealed in [29]. Therefore, the current distributions shown in the present paper are valid at lower frequencies. The frequency dependence of current distributions will be investigated in the near future using spectral-domain analysis with a large number of basis functions for both single and coupled microstrip lines on isotropic and anisotropic substrates.

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